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STATIONARY QUASITRANSVERSE SIMPLE AND SHOCK WAVES IN A WEAKLY ANISOTROPIC NON-LINEAR ELASTIC MEDIUM*

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Two-dimensional stationary simple and shock waves in a weakly anisotropic non-linear elastic medium are considered under the same assumptions as in /1-6/, which studied one-dimensional non-stationary simple and shock waves in a prestrained non-linear elastic medium.

The standard analysis of stationary simple and shock waves /7-9/ in the magnetohydrodynamics of a gas with a frozen magnetic field essentially corresponds to a special case of an anisotropic elastic medium. Particular plane selfsimilar boundary-value problems of shock wave reflection from the boundary of an isotropic non-linear elastic half-space were solved numerically in /9, 10/.

1. Equations describing the behaviour of two-dimensional stationary simple waves. A weakly anisotropic non-linear elastic medium is defined by the elastic potential /1/

$$\Phi = \rho_0 U(\varepsilon_{ij}, p_l^{(k)}\ldots, \ldots, S), \quad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial w_i}{\partial \eta_j} + \frac{\partial w_j}{\partial \eta_i} + \frac{\partial w_k}{\partial \eta_i}\frac{\partial w_k}{\partial \eta_j}\right)$$

Here U is the internal energy of the medium, S is the entropy per unit mass, ε_{ij} are the components of Green's strain tensor, ρ_0 is the density in the unstressed state, $p_{l\ldots}^{(k)}$ are tensors specifying the deviation of the medium from an isotropic medium, w_i is the displacement vector and η_i are the Lagrangian coordinates (Cartesian right coordinates in the unstressed state); here and henceforth, $i, j, k = 1, 2, 3$.

The system of three equations of motion in Lagrangian Cartesian variables has the form /2/

$$\rho_0 \frac{\partial^2 w_i}{\partial t^2} = \frac{\partial}{\partial \eta_j}\frac{\partial \Phi}{\partial(\partial w_i/\partial \eta_j)} \tag{1.1}$$

and is of hyperbolic type.

We introduce a moving system of coordinates ξ_1, ξ_2, ξ_3 in which the motion of the system is steady.

$$\xi_1 = \eta_1 - |\mathbf{W}|t\sin\alpha, \ \xi_2 = \eta_2 - |\mathbf{W}|t\cos\alpha, \ \xi_3 = \eta_3$$

where $\mathbf{W}$ is a given vector of sufficiently large absolute value. The angle α defines the direction of the vector $\mathbf{W}$ relative to the axes η_1, η_2, η_3.

Let

$$\partial w_i/\partial \xi_1 = l_i, \ \partial w_i/\partial \xi_2 = m_i, \ \partial w_i/\partial \xi_3 = a_i$$

We assume that l_i, m_i, a_i are functions of the two variables ξ_1 and ξ_2. Therefore, we see from the equalities

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$$\partial a_i/\partial \xi_1 = \partial l_i/\partial \xi_3 = 0, \partial a_i/\partial \xi_2 = \partial m_i/\partial \xi_3 = 0$$

that a_i are constant ($a_i = \text{const}$) and may occur in the description of the anisotropic properties of the medium as parameters.

Using the above notation, we will rewrite the equations of steady motion in the coordinate system ξ_1, ξ_2, ξ_3 in the form

$$\rho_0 W^2\left(\sin^2\alpha \frac{\partial l_i}{\partial \xi_1} + \cos^2\alpha \frac{\partial m_i}{\partial \xi_2} + 2\sin\alpha\cos\alpha \frac{\partial m_i}{\partial \xi_1}\right) = r_{ik}\frac{\partial l_k}{\partial \xi_1} + d_{ik}\frac{\partial m_k}{\partial \xi_1} + d_{ik}^T \frac{\partial l_k}{\partial \xi_2} + s_{ik}\frac{\partial m_k}{\partial \xi_2} \tag{1.2}$$

$$\frac{\partial l_k}{\partial \xi_2} = \frac{\partial m_k}{\partial \xi_1}, \quad r_{ik} = \frac{\partial^2 \Phi}{\partial l_i \partial l_k}, \quad d_{ik} = \frac{\partial^2 \Phi}{\partial l_i \partial m_k}, \quad s_{ik} = \frac{\partial^2 \Phi}{\partial m_i \partial m_k}$$

(d_{ik}^T is the transpose of the matrix d_{ik}). The quantities r_{ik}, d_{ik}, s_{ik}, d_{ik}^T are computed using the given elastic potential Φ. For a weakly anisotropic elastic medium, the potential Φ can be represented as the sum of two terms $\Phi = \Phi_0 + \Phi_1$ /1/. The first term describes an isotropic non-linear elastic medium without initial strains. The small second term describes the deviation of the internal energy of the material from the isotropic energy. This term is a function of convolutions of the strain tensor with anisotropy tensors. We expand Φ_0 and Φ_1 in powers of l_i and m_i:

$$\Phi_0 = {}^1\!/_2 \lambda I_1^2 + \mu I_2 + \beta I_1 I_2 + \gamma I_3 + \nu I_1 + \xi I_2^2 + \ldots + \rho_0 T_0 (S - S_0) \tag{1.3}$$

$$I_1 = \varepsilon_{ii}, \ I_2 = \varepsilon_{ij}\varepsilon_{ij}, \ I_3 = \varepsilon_{ij}\varepsilon_{jk}\varepsilon_{ki}$$

$$\Phi_1 = B_1 l_1^2 + B_2 l_2^2 + B_3 l_3^2 + B_4 l_1 l_2 + B_5 l_1 l_3 + B_6 l_2 l_3 + B_7 m_1^2 + B_8 m_2^2 + B_9 m_3^2 + B_{10} m_1 m_2 + B_{11} m_1 m_3 + B_{12} m_2 m_3$$

Here $\lambda, \mu, \beta, \gamma, \nu$ and ξ are the elastic moduli of the medium and B_i are constants associated with the anisotropy of the material. We shall assume that B_i are of the order of δ ($\delta \ll 1$ is the anisotropy parameter).

For the system of Eqs.(1.2) we will seek a solution in the form of a two-dimensional stationary simple wave, i.e., such that $l_i = l_i(\theta(\xi_1, \xi_2))$, $m_i = m_i(\theta(\xi_1, \xi_2))$, $a_i = \text{const}$ (θ is a function of its arguments), and the entropy S is constant ($\partial S/\partial \xi_i = 0$). This leads to a system of ordinary differential equations for l_i and m_i (δ_{ik} is the Kronecker delta)

$$(C_1^2\delta_{ik} - b_{ik}) dl_k/d\theta = 0, \ dm_k/d\theta = -\Psi dl_k/d\theta \tag{1.4}$$

$$(\Psi = -(\partial\theta/\partial\xi_2)/(\partial\theta/\partial\xi_1) = -(d\xi_1/d\xi_2)_\theta$$

$$C_1^2 = C - 2\rho_0 W^2 \sin\alpha\cos\alpha\,\Psi + \rho_0 W^2\cos^2\alpha\,\Psi^2$$

$$C = \rho_0 W^2\sin^2\alpha, \ b_{ik} = r_{ik} - (d_{ik} + d_{ik}^T)\Psi + s_{ik}\Psi^2)$$

Let us fix α by a special choice of the axes η_1, η_2, η_3 so that in the corresponding axes ξ_1, ξ_2, ξ_3 we have the equality $\Psi = 0$ on the first characteristic. This choice of the axes is possible if the determinant of system (1.4), treated as a polynomial in Ψ, has at least one real root.

If the elastic medium is linear, the coefficients of the system of Eqs.(1.4) are constant and the equality $\Psi = 0$ therefore holds on all the characteristics of the system of Eqs.(1.2). Then, first, we see from the second equation in (1.4) that the quantities m_i are constant in the wave. Second, noting that $\partial\theta/\partial\xi_2 = 0$, we conclude that l_i depend only on a single variable ξ_1, and therefore the direction of wave propagation is known (the normal to the characteristics - the axis ξ_1 - is known). The system of six Eqs.(1.4) reduces to a system of three equations for l_i,

$$(C^2\delta_{ik} - r_{ik}) dl_k/d\theta = 0$$

The condition for the determinant of this system to be zero is a cubic equation in C^2 and for a given W it is used to find the angle α. Note that the restrictions on α are a consequence of the previous requirement $\Psi = 0$. The quantity C is the characteristic velocity relative to the medium when the normal to the characteristic is in the direction of the ξ_1 axis. Since $r_{ik} = \text{const}$, we have $C = \text{const}$.

If the medium is linear and isotropic, the matrix $\|r_{ik}\|$ has the form $r_{11} = \lambda + 2\mu$, $r_{22} = r_{33} = \mu$, $r_{ij} = 0$ for $i \neq j$. For transverse waves $\alpha = \arcsin\sqrt{\mu/\rho_0 W^2}$, and for longitudinal waves $\alpha = \arcsin\sqrt{(\lambda + 2\mu)/\rho_0 W^2}$.

If the elastic medium is weakly anisotropic and weakly non-linear, then $\Psi(l_1, l_2, l_3, m_1, m_2, m_3)$ is small and the square of the characteristic velocity relative to the medium C_1^2 varies. The wave does not have a unique fixed direction of propagation (the normal to the characteristic is variable), but due to the smallness of non-linearity we may assume that the main variation of the parameters in a simple wave is in the direction of the ξ_1 axis, i.e., l_i are essentially variable in the wave, while m_i do not change much (this will be proved below).

2. Two-dimensional stationary simple quasitransverse waves. Quasitransverse waves are waves in which the ratio of the change in the longitudinal component (l_1) and the transverse component (l_2, l_3) is a small quantity of the order of the initial shear strain /3/.

Let the strains l_i produced by the passage of waves in the medium be small quantities of order not exceeding ε. Then $\Psi = O(\varepsilon^2)$, because the change in the characteristic velocity relative to the medium $\Delta C^2 = C_1^2 - C_1$ is of the order of $\varepsilon^2/2$, 3/ and Ψ is related to the change in the characteristic velocity. From the second equation in (1.4) we see that the order of change of m_i is ε^3. We introduce the smallness parameter $\chi = \max\{\varepsilon^2, \delta\}$, where $(\delta \ll 1$ is the anisotropy parameter /4/, and in the system of Eqs.(1.4) we allow only for terms of order not exceeding $\varepsilon\chi$. Then the three equations for l_i in system (1.4) can be solved independently. The quantities m_i, together with a_i, may be used to describe the anisotropic properties of the medium.

For the variable Ψ we obtain the equation

$$|C_1^2\delta_{ik} - b_{ik}| = 0 \tag{2.1}$$

For quasitransverse waves, we can eliminate the longitudinal component l_1 by expressing it approximately in terms of the components l_2 and l_3, as in /4/. To prove this statement, let

$$b_{ik} = b_{ik}^\circ + g_{ik}$$

$$g_{ik} = \frac{\partial^2 P}{\partial l_i \partial l_k} - \left(\frac{\partial^2 P}{\partial l_i \partial m_k} + \frac{\partial^2 P}{\partial m_i \partial l_k}\right)\Psi + \frac{\partial^2 P}{\partial m_i \partial m_k}\Psi^2$$

$$P = \Phi - {}^1/_2(\lambda + 2\mu) l_1^2 - {}^1/_2\mu(l_2^2 + l_3^2)$$

where b_{ik}° is the matrix corresponding to a linear isotropic medium. From (1.4) for $i = 1$ we obtain

$$C_1^2 \partial l_1/\partial\theta = b_{1k}\partial l_k/\partial\theta \tag{2.2}$$

For quasitransverse waves we may approximately take $C_1^2 = \mu + O(\chi)$, because, first, Ψ is small and, second, $C^2 = \mu$ for transverse waves. Then from (2.2) we have

$$\mu\frac{\partial l_1}{\partial\theta} = (\lambda + 2\mu)\frac{\partial l_1}{\partial\theta} + g_{12}\frac{\partial l_2}{\partial\theta} + g_{13}\frac{\partial l_3}{\partial\theta}$$

$$l_1 = -\frac{1}{\lambda + \mu}\frac{\partial P}{\partial l_1} + l_1^\circ, \quad l_1^\circ = \text{const}$$

where the superscript $^\circ$ relates to the state in front of the wave. Using this equality, we rewrite the equations of motion (1.4) in the form

$$C_1^2\frac{\partial l_\beta}{\partial\theta} = (\mu\delta_{\beta\gamma} + h_{\beta\gamma})\frac{\partial l_\gamma}{\partial\theta} \equiv \frac{\partial}{\partial\theta}\frac{\partial Q}{\partial l_\beta}, \quad \beta, \gamma = 2, 3 \tag{2.3}$$

$$h_{\beta\gamma} = g_{\beta\gamma} - \frac{1}{\lambda + \mu}\overset{\circ}{g}_{1\beta}\overset{\circ}{g}_{1\gamma} = \frac{\partial F}{\partial l_\beta \partial l_\gamma}$$

$$Q = \frac{\mu(l_2^2 + l_3^2)}{2} + F(l_2, l_3), \quad F = P^\circ - \frac{1}{2(\lambda + \mu)}\left(\frac{\partial P}{\partial l_1}\right)^\circ$$

This system of equations contains two equations for l_2 and l_3. For a medium with anisotropic (1.3), the two-dimensional potential can be represented in the form /1/

$$F(l_2, l_3) = {}^1/_2(f + g) l_2^2 + {}^1/_2(f - g) l_3^2 - {}^1/_8\varkappa_1(l_2^2 + l_3^2)^2 + s_1 l_2 l_3$$

$$\left(f = \mu + B_2 + B_3 - \frac{B_4^2 + B_5^2}{2(\lambda + \mu)}, \quad g = B_2 - B_3 + \frac{B_5^2 - B_4^2}{2(\lambda + \mu)},\right.$$

$$\left. s_1 = B_6 - \frac{B_5 B_4}{\lambda + \mu}\right)$$

The change of state in the wave may be demonstrated in the $l_2 l_3$ plane. Therefore, by rotating the coordinate axes in this plane, we can eliminate the term $s_1 l_2 l_3$ in the expression for the elastic potential F /1/. The potential F thus takes the form

$$F = {}^1/_2\,(f + g^*)\, l_2^{*2} + {}^1/_2\,(f - g^*)\, l_3^{*2} - {}^1/_8\,\varkappa_1\,(l_2^{*2} + l_3^{*2})^2$$

where $g^* = (g^2 + s_1^2)^{1/2}$, $l_2^* = -l_3 \sin\varphi + l_2 \cos\varphi$

$$l_3^* = l_3 \cos\varphi + l_2 \sin\varphi, \quad \operatorname{tg} 2\varphi = -s_1 g$$

The asterisk will be omitted in what follows.

The constants $f, g, \varkappa_1$ have the following physical meaning: the small quantity g is the anisotropy parameter, f is the characteristic velocity when there is no non-linearity and anisotropy and $\varkappa_1$ is the elastic constant characterizing the non-linear properties of the medium in quasitransverse waves.

For the simplified system of Eqs.(2.3), condition (2.1) is rewritten in the form

$$\left\| \begin{matrix} h_{22} - C_1^2 & h_{23} \\ h_{33} & h_{33} - C_1^2 \end{matrix} \right\| = 0; \quad h_{\beta\gamma} = \frac{\partial^2 F}{\partial l_\beta \partial l_\gamma}, \quad \beta, \gamma = 2, 3$$

Solving the quadratic equation for Ψ, we obtain

$$\Psi_{1,2} = A^{-2}\,(C^2 - f + \varkappa_1\,(l_2^2 + l_3^2 \pm {}^1/_2\,((l_3^2 - l_2^2 + 2g\varkappa_1^{-1})^2 + 4l_2^2 l_3^2)^{1/2})) \qquad (2.4)$$

We see from (2.4), that Ψ_1 and Ψ_2 differ from each another by a quantity of the order of χ. The indexing of Ψ_i is such that $\Psi_1 > \Psi_2$ and we correspondingly distinguish between fast quasitransverse waves (Ψ_1) and slow quasitransverse waves (Ψ_2).

The differential equations for the integral curves of quasitransverse stationary simple waves are obtained from the system of Eqs.(2.2) using (2.4):

$$\frac{dl_2}{dl_3} = \frac{l_2^2 - l_3^2 - 2g\varkappa_1^{-1} \mp ((l_2^2 - l_3^2 - 2g\varkappa_1^{-1})^2 + 4l_2^2 l_3^2)^{1/2}}{2 l_2 l_3} \qquad (2.5)$$

Eqs.(2.5) are identical with the differential equations of quasitransverse non-stationary one-dimensional simple waves /3/. The integral curves of quasitransverse stationary two-dimensional simple waves are therefore identical with the integral curves of quasitransverse non-stationary one-dimensional simple waves investigated in /3/.

Eqs.(2.5) represent in the $l_2 l_3$ plane two mutually orthogonal families of integral curves for the fast and the slow quasitransverse waves. These families of integral curves are shown in Fig.1. The curves have two singular points $l_3 = 0$ and $l_2 = \pm\sqrt{2g\varkappa_1^{-1}}$ on the l_2 axis. Both families are symmetrical about the coordinate axes. The quantity g has a considerable effect on the form of the integral curves. As we move away from the origin $(l_2^2 + l_3^2 \gg 2g\varkappa_1^{-1})$, the integral curves of one family approach circles centred at the origin and the integral curves of the other family approach rays. If $g = 0$, then all the curves become circles and rays. By (2.4), for media with $\varkappa_1 > 0$, ovals are the integral curves of fast waves and lines that go to infinity are the integral curves of slow simple waves. For materials with $\varkappa_1 < 0$, the situation is reversed.

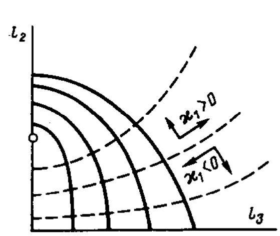

Fig.1

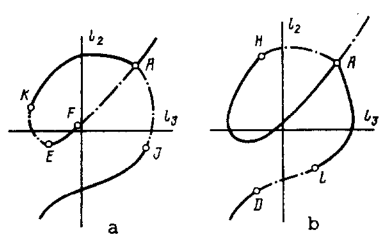

Fig.2

If the initial strain is of the order of ε, the integral curves are two orthogonal families of curves parallel to the l_2 and l_3 axes. This pattern of integral curves is observed in the linear anisotropic case /1, 3/.

If the rectilinear characteristics of the wave issue from a common point $(\xi_1^\circ, \xi_2^\circ)$, the wave is called centred. Centred flows are selfsimilar: the parameters of the medium in centred flows depend only on the ratio ξ_1/ξ_2 for an appropriate choice of the origin. The criteria for the existence of a continuous selfsimilar solution in the form of a quasi-transverse stationary simple wave (the existence of $\Psi < 0$) are identical with the non-overturning criteria for one-dimensional non-stationary simple waves derived in /3/.

For $\varkappa_1 > 0$ a continuous selfsimilar solution exists in the form of fast waves with decreasing $|l_3|$ and slow waves with increasing $|l_3|$ (Fig.1). For $\varkappa_1<0$, a continuous solution exists in the form of fast and slow waves with decreasing $|l_3|$ (Fig.1).

3. ***Quasitransverse shock waves.*** For any dynamic quantity a, the jump across a discontinuity will be denoted by $[a]=a^+-a^-$, where a^+ and a^- is the value of a behind and in front of the discontinuity.

The conditions on the discontinuity are described in the same way as in /2, 5/, expressing conservation of momentum and energy:

$$\rho_0 W^2 \sin^2\alpha_1 [l_k] = \left[\frac{\partial\Phi}{\partial l_k}\right] \tag{3.1}$$

$$[\Phi] = \frac{1}{2}\left[\frac{\partial\Phi}{\partial l_k}\right][l_k] + \left(\frac{\partial\Phi}{\partial l_k}\right)^+ [l_k]$$

Note that in previous studies /2, 5, 6/ the direction of propagation of the discontinuity was known (the normal to the discontinuity was known) and the dynamic conditions were written in the system of coordinates attached to the discontinuity plane. The study of discontinuities in this paper is more complicated because the discontinuity does not have a fixed direction of propagation (the normal to the discontinuity is unknown). The angle α_1 between the direction of a vector and the discontinuity plane may be represented in the form $\alpha_1=\alpha+\Psi$. The quantity α can be determined in the linear approximation. The quantity Ψ is variable and is of order ε^2. The quantities l_1°, l_2°, l_3°, m_1°, m_2°, m_3° defining the initial state of the medium in the coordinate system associated with the angle α change by a quantity of the order of ε^2 on changing to a coordinate system associated with the angle α_1.

The conditions on the discontinuity may be written in explicit form. To this end we expand the function Φ in powers of $[l_k]$ and $[S]$ so that

$$\Phi = \Phi_0 + \left(\frac{\partial\Phi}{\partial l_k}\right)[l_k] + \rho_0 T_0 [S] + \Phi_1([l_k])$$

The function Φ_1 is the expansion of Φ in powers of $[l_k]$ starting with the second and higher degree, with coefficients that depend on the state in front of the discontinuity. The function Φ_1 is chosen so that $[\partial\Phi/\partial l_k]=\partial\Phi_1/\partial[l_k]$. Then the conditions on the discontinuity (3.1) are written in terms of the function Φ_1 in the form /5, 6/

$$(\rho_0 W^2\sin^2\alpha - 2\rho_0 W^2 \sin\alpha\cos\alpha\Psi)[l_k] = \frac{\partial\Phi_1}{\partial[l_k]} \tag{3.2}$$

$$\rho_0 T_0 [S] = \frac{1}{2}\frac{\partial\Phi_1}{\partial[l_k]}[l_k] - \Phi_1 \tag{3.3}$$

The function Φ_1 does not contain the entropy, and Eqs.(3.2) therefore relate the quantities $[l_1]$, $[l_2]$, $[l_3]$ and Ψ. If we eliminate Ψ, the result is the equation of the shock polar. In the space of displacement gradients l_1, l_2, l_3, the shock polar is the set of states l_1, l_2, l_3 which can be reached by a jump from the initial state L_1, L_2, L_3 without violating the laws of conservation. Condition (3.3) is used to compute the entropy at the jump, and its right-hand side must therefore be non-negative.

Quasitransverse waves are waves for which $[l_1]\ll\max([l_2],[l_3])$. From the jump conditions (3.2) and (3.3) (for quasitransverse waves these conditions relate the quantities $[l_2]$, $[l_3]$ and Ψ), we obtain

$$\Psi = A^{-2}\Big(C^2 - f + {}^1/_2\varkappa_1\Big\{l_2^2+l_3^2-R^2+L_2l_2+L_3l_3+$$
$$2\,\frac{g\varkappa_1^{-1}(l_3-L_3)^2 - g\varkappa_1^{-1}(l_2-L_2)^2 + 2[(l_2-L_2)L_2+(l_3-L_3)L_3]^2}{(l_2-L_2)^2+(l_3-L_3)^2}\Big\}\Big)$$
$$R^2 = L_2^2 + L_3^2$$

and the equation of the shock polar

$$(l_2^2+l_3^2-R^2)(L_3l_2-L_2l_3) + 2g\varkappa_1^{-1}(l_2-L_2)(l_3-L_3) = 0$$

which is identical with the equation of the shock adiabatic of quasitransverse shock waves /5, 6/. The conditions of evolution and non-decreasing entropy are similar to the corresponding conditions derived in /5, 6/.

The shock polar is shown in Fig.2, where the sections of the shock polar satisfying the conditions of non-decreasing entropy and evolution are shown by the dash-dot curves for media with $\varkappa_1>0$ in Fig.2a and for media with $\varkappa_1<0$ in Fig.2b.

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STRESS-STRAIN STATES IN A MULTISHEET SURFACE WITH CUTS*

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The first, second and mixed fundamental boundary-value problems of elasticity theory are considered on an n-sheet Riemann surface with straight-line cuts joining the branch points. The cuts are such that their edges are situated in different planes. Complex potentials are constructed, asymptotic representations of the stresses and derivatives of the displacement components are obtained near the vertices of the cuts and invariant Γ- integrals /1/ are obtained, by the method of reduction to a matrix Riemann boundary-value problem.

The first and second fundamental problems for an $n=2$ Riemann surface were solved /2/ by the Riemann boundary-value problem method for a Riemann surface. For $n=1$ the results are identical with previously known results for a plane /3/.

1. Statement of the problem. Suppose we have n identical thin homogeneous isotropic elastic infinite plates $E_1, E_2, \ldots, E_n$ of the same thickness and with cuts along the same intervals $l_j=[a_j, b_j]$ $(j=1, 2, \ldots, m)$ along the real x axis superimposed on one another so that, for all the plates, cuts with the same numbers are placed above one other. The lower edges of the plate E_k are glued to the corresponding upper edges of plate E_{k+1} $(k=1, 2, \ldots, n-1)$. The upper edges of the cuts of E_1 and the lower edges of E_n are not glued together. We shall denote them by L^+ and L^- respectively. If one takes a section perpendicular to

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